RAN - 2003000204020033

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S. Y. B. Sc. (Sem. - IV) Examination April - 2023

Mathematics : Paper - 403

Introduction to Abstract Algebra

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S. Y. B. Sc. (Sem. - IV)**

Name of the Subject :

☛ **Mathematics : Paper - 403 Introduction to Abstract Algebra**

Subject Code No.: **2003000204020033**

Seat No.:

Student's Signature

Q. 1. Answer any Five from the following: **[10]**

1. If $a \mid b$ and $b \mid a$ then prove that $a = \pm b$ for any $a, b \in Z$.
2. For any $a, b \in Z$, define $gcd(a, b)$ and $lcm(a, b)$.
3. In a group G, prove that the identity element of G is unique.
4. Show that the set of all even integers is a subgroup of Z with respect to addition.
5. Define an abelian group and give an example of non-abelian group.
6. Define the order of a group with illustration.
7. If R is a ring, then for all $a, b \in R$, prove that $(-a)(-b) = (-ab)$.
8. Which of the rings Z_7 and Z_8 are fields? Why?

Q. 2. **Answer any Two from the following:** **[10]**

1. For $a, b, c \in Z$ and n is any nonnegative integer,
   If $a \equiv b\,(mod\ n)$, $c \equiv d\,(mod\ n)$ then prove that $a + c \equiv b + d\,(mod\ n)$ and $ac \equiv bd\,(mod\ n)$.

2. Find integers m and n satisfying $(1128, 33) = 1128m + 33n$ and also find [1128,33].

3. For any $a, b, c, m, n \in Z$ if $a \mid b$ and $a \mid c$ then prove that $a \mid mb + nc$ and $a \mid b^2 - c^2$.

Q. 3. **Answer any Two from the following:** **[10]**

1. State and prove necessary and sufficient condition for a nonempty subset H of a group G to be a subgroup of the group G.

2. Show that the set $G = \{-1.1\}$ is an abelian group under multiplication. What is the order of G?

3. If G is a group then prove that $(ab)^{-1} = b^{-1}a^{-1}$; $\forall\ a, b \in G$.

Q. 4. **Answer any Two from the following:** **[10]**

1. Define the center of the group and prove that it is a subgroup of the group.

2. Prove that intersection of two subgroups of a group is also a subgroup. What can you say about union of two subgroup?

3. Prove that every cyclic group is abelian.

Q. 5. **Answer any Two from the following:** **[10]**

1. Prove that J_p, the ring of integers modulo p is a field if p is prime.

2. Define Integral domain. Prove that a finite integral domain is field.

3. Prove that every Boolean Ring is commutative.

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